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September 5, 2023

51. Mass Balance Module

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51.1. Where do we need mass balancing?

Mass balancing is a common practice in metallurgy. The mass balance of a circuit is needed for several reasons:

1. To estimate the metallurgical performance of the circuit
2. To locate process bottlenecks and for circuit diagnosis
3. To create models of the processing stages
4. To simulate the process.

The following steps are often required to simulate a process:

1. Collecting experimental data (experimental work, sampling, sample preparation, assaying)
2. Mass balancing and data reconciliation of the experimental data
3. Model building
4. Simulation.

In HSC Chemistry, you can do all the steps in one program: *HSC Sim* with the *Mass Balance* tool. The work flow in the HSC Sim and Mass Balance tool starts from a flowsheet drawing, followed by importing experimental data, performing mass balancing, model fitting & building (model fitting will be available later on) and simulation (**Fig. 1**).

Fig. 1. From flowsheet with data through mass balancing to modeling and simulation.

51.2. Mass balancing capabilities in HSC Chemistry

HSC allows the user to solve the following mass balance problems (**Table 1**). There are three different possibilities for the solution:

- Solids
- Assays only
- Solids and water independently

Liquid flow rates and solids percentage are solved only if Solve solids and water independently is chosen. If Assays only is chosen, there must be a Total solids measurements for all the flows to be balanced.

1. **Unsized**
   - Balance Total Solids flow rates. At least one total solids flow rate measurement must be given
   - Balance Total Water/liquid flow rate. This is done by first balancing the Total solids flow rates and after that the Total Water/liquid flow rate independently
   - Balance solids percentage
   - Balance Solids component assays and/or Mineral assay.

2. **Sized**
   - Balance Total Solids flow rates for bulk and all the size fractions. At least one total solids flow rate measurement for bulk must be given
   - Balance particle size fraction-% assays
   - Balance Total Water/liquid flow rate for bulk. This is done by first balancing the Total solids flow rates and after that the Total Water/liquid flow rate independently
   - Balance solids percentage for bulk
   - Balance Solids component assays/Mineral assay for bulk.

3. **Sized by assay**
   - Balance Total Solids/Slurry flow rates for bulk and all the size fractions. At least one total solids/slurry flow rate measurement must be given
   - Balance particle size fraction-% assays
   - Balance Total Water/liquid flow rate for bulk. This is done by first balancing the Total solids flow rates and after that Total Water/liquid flow rate independently
   - Balance solids percentage for bulk

   Balance Solids component assays/Mineral assay for bulk and the size fractions.

Table 1. Mass balance cases that can be solved with HSC Sim.

Process Data	Balanced Unsized (Bulk)	Balanced Sized
Flow rates	✓ Stream mass flow rates	✓ Fraction mass flow rates
Water (liquid)	✓ Stream liquid mass flows	
Assays (species, minerals)	✓ Bulk composition	✓ Composition by size
Particle Sizes		✓ Fraction-%
Solids-%	✓ Solids-%	

In addition to above HSC calculates:

✓ Recoveries of all solids and liquid components, unsized (bulk) and sized

✓ For mineral balances: specific gravities (SG) of stream solids, unsized (bulk) and sized

✓ For mineral & solids-% balances: stream volumetric flow rates

51.3. Overview of the HSC Sim Mass Balancing tool

The HSC Mass Balance tool is started from the HSC Main Menu dialog or from HSC Sim Menu: Tools → Mass Balance. The window layout consists of: Balancing Navigator, Working Area, Property Panel, and Upper Buttons.

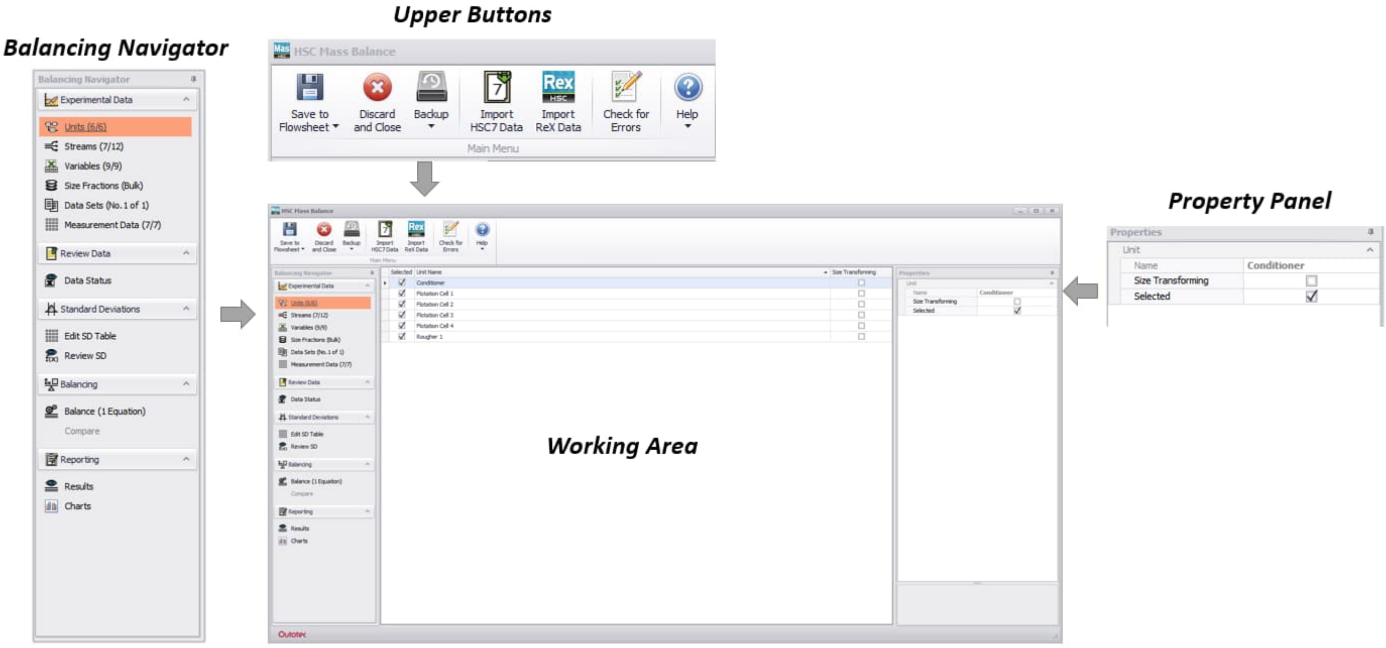


Fig. 2. Main components of the Mass Balance window.

51.4. Step-by-step example: data reconciliation and mass balancing in HSC

This step-by-step example shows how to do mass balancing for unsized data. The work starts from HSC Sim by drawing the process flowsheet. The Mass Balance tool is started from the HSC Sim Tools menu. The steps to import the data and to balance it follow the left side Balancing Navigator panel (**Fig. 2**) from top to bottom. More examples are found in Chapter 52.

51.4.1. Drawing the flowsheet

First the flowsheet is prepared in HSC Sim. In this example it looks as shown in **Fig. 3**.

HSC will create the mass balance equations according to the available data, and multiple data sets can be created. Therefore, there is no need to draw a flowsheet for mass balancing only or a new flowsheet every time for different kinds of mass balance problems.

When naming the streams, it is a good idea to use identical names to those in your analysis data file. Before proceeding, please check the stream connections and check the flowsheet for possible errors.

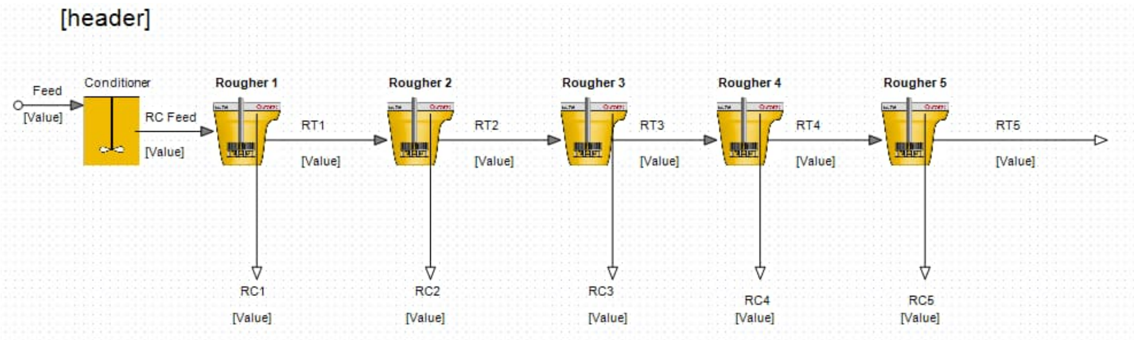


Fig. 3. HSC Sim flowsheet drawing of a flotation process with rougher cells.

51.4.2. Importing the experimental data

When the flowsheet is ready (i.e. all streams are named properly and connections have been checked), you can import your experimental data. The following subsections will concentrate on how to import the data for mass balancing and data reconciliation.

1. Select units

The units are listed based on the flowsheet drawing figure, but they cannot be edited here.
In this view you can:

- Select or deselect the units to be included in the balancing calculation.
- Set whether the unit will transform the particle size distribution of the solids, e.g. grinding mills. This selection is needed in sized balancing, to indicate that the fraction balance will not be held over those units. For bulk balances, this has no effect.

Selected	Unit Name	Size Transforming
☑	Conditioner	☐
☑	Rougher 1	☐
☑	Rougher 2	☐
☑	Rougher 3	☐
☑	Rougher 4	☐
☑	Rougher 5	☐

Fig. 4. Experimental Data – Units.

2. Select streams

The streams are listed based on the flowsheet drawing figure, the stream names can be edited only in the Sim flowsheet.

Here you can:

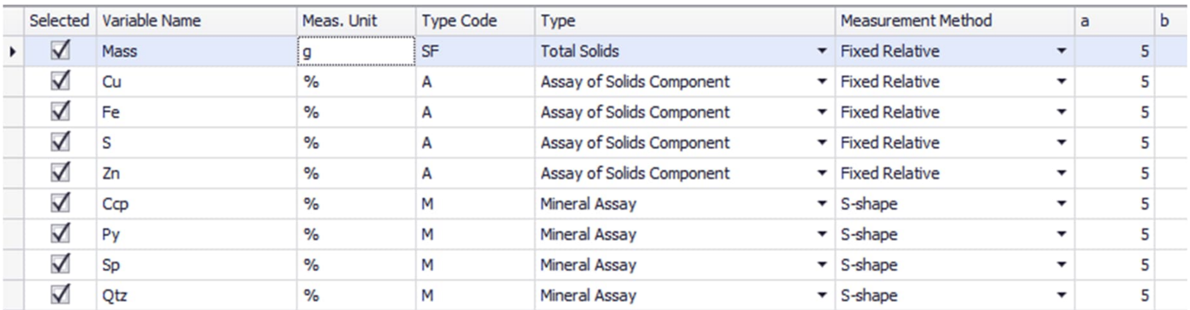
- Select and deselect the streams to be included in the balancing calculation.
- Change the stream type:
    - Unknown (HSC will detect the type automatically after importing the data)
    - Solids/Slurry
    - Liquid/Water
- Select and deselect some specific flow or assay from the balance calculation
- Set stream sampling error based on sampling device and sample quality
- Streams can be reordered up/down on the list, the measurement data table is rearranged according this as well

Selected	Stream Name	Type	Flows	Assays	Sampling Device	Sample Quality
☑	Feed	Solids/Slurry		☑	Default	Good
☑	RT1	Solids/Slurry			Default	Good
☑	RT2	Solids/Slurry			Default	Good
☑	RT3	Solids/Slurry			Default	Good
☑	RT4	Solids/Slurry			Default	Good
☑	RT5	Solids/Slurry	☑	☑	Default	Good
☑	RC1	Solids/Slurry	☑	☑	Default	Good
☑	RC2	Solids/Slurry	☑	☑	Default	Good
☑	RC3	Solids/Slurry	☑	☑	Default	Good
☑	RC4	Solids/Slurry	☑	☑	Default	Good
☑	RC5	Solids/Slurry	☑	☑	Default	Good
☑	RC Feed	Solids/Slurry			Default	Good

Fig. 5. Experimental Data – Streams.

3. Add variables

Next, the measured variables are added / removed from the upper bar buttons. The variable name and unit on the list can be edited, measurements methods and their parameters can be edited and the variable can also be unselected from balancing if desired. The variable types are listed in **Table 2**.



Selected	Variable Name	Meas. Unit	Type Code	Type	Measurement Method	a	b
☑	Mass	g	SF	Total Solids	Fixed Relative	5	
☑	Cu	%	A	Assay of Solids Component	Fixed Relative	5	
☑	Fe	%	A	Assay of Solids Component	Fixed Relative	5	
☑	S	%	A	Assay of Solids Component	Fixed Relative	5	
☑	Zn	%	A	Assay of Solids Component	Fixed Relative	5	
☑	Ccp	%	M	Mineral Assay	S-shape	5	
☑	Py	%	M	Mineral Assay	S-shape	5	
☑	Sp	%	M	Mineral Assay	S-shape	5	
☑	Qtz	%	M	Mineral Assay	S-shape	5	

Fig. 6. Experimental Data – Variables.

Table 2. Data types of the variables.

Data type	Abbreviation	Examples	Meas. Units
Total solids flow rate	SF	Total t/h, Mass g	Any mass unit (t/h default)
Solids component flow rate	SC	Iron t/h, plastic t/h, gold g	Any mass unit (t/h default)
Assay of solids component	A	Cu%, P2O5%, Au g/t	%, g/t, ppm
Mineral assay	M	Ccp%, Py%, Qtz%, Au g/t	%, g/t, ppm
Size fraction assay	SA	0-20um %, 20-45um %	%
Solids percentage	SP	35%	%
Info		Column with comments, extra data (temperature), sampling notes, etc.	-

4. Add size fractions

By default, one size fraction exists: 'Bulk', which cannot be removed, but can be renamed. More size fractions can be added to / removed from the list by clicking the buttons on the upper bar. Fraction names can be given freely.

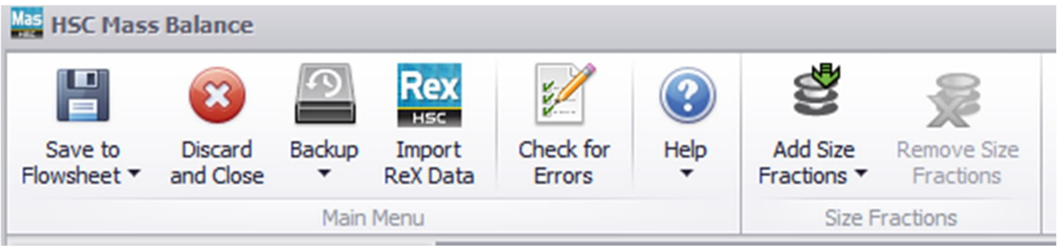


Fig. 7. Experimental Data – Size Fractions.

5. Add dataset(s)

By default, one dataset exists, which can be renamed here. In addition, more datasets can be added and selected one at a time to carry out data reconciliation. Clone Data Set button will create a replica of an existing data set with all balancing settings.

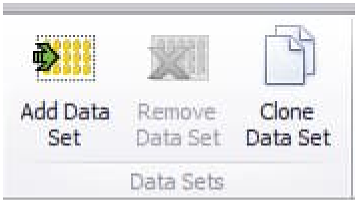
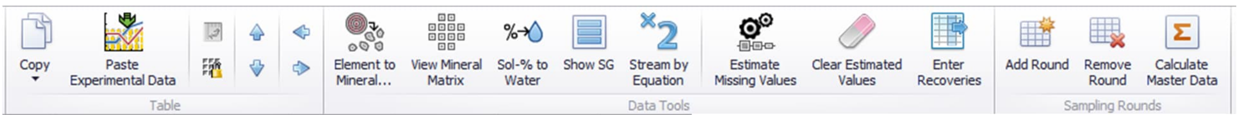


Fig. 8. Experimental Data – Datasets.

6. Import measurement data

In this view, a stream & variable template is automatically generated and displayed. The data can be entered by:

a) Typing manually in the table
b) Copying an empty data template → organizing the data e.g. in Excel → select 'Paste Experimental Data'. This will automatically place the data in the correct rows and columns based on the clipboard table content. Note: if the data is horizontal (stream names in rows), HSC will automatically identify the stream and variable names in order to place the data correctly.
c) Importing the old HSC7 Analyses.xls mass balance file
d) You can also import <u>measured recoveries</u> for model fit in this view by pressing enter recoveries button and the pasting the data. After you have done this press save to flow sheet and close mass balance and open model fit.



	B	C	D	E	F	G
1	Stream	Mass g	Cu %	Fe %	S %	Zn %
2	Feed	15097	1.013	36.3	41.3	1.963
3	RT1	14632.8	0.337	34.398	39.524	1.952
4	RT2	14514.4	0.221	33.388	38.573	1.953
5	RT3	14347.7	0.099	32.376	37.617	1.933
6	RT4	14140.1	0.069	31.310	36.628	1.929
7	RT5	13899	0.0820	30.20	35.6	1.960
8	RC1	464	23.60	29.20	34.4	5.78
9	RC2	118.4	17.00	28.90	34.9	8.56
10	RC3	166.7	12.50	28.60	34.8	8.45
11	RC4	207.6	3.56	31.20	35.9	6.07
12	RC5	241.1	0.560	31.8	36.4	3.44
13	RC Feed	15097	1.033	35.252	40.317	2.016

Fig. 9. Experimental Data – Measurement Data.

In addition, here you can:

- ✓ Do element to mineral calculation for mineral based balancing
- ✓ Do Solids-% to water calculation, and show them visible in the table (anyhow, HSC will do that automatically for water balancing)
- ✓ Estimate missing values, and show them visible in table. **NOTE: Estimations of missing values isn't compulsory**. As a rule of thumb: it is *not recommended* at first. It is only needed in cases when the combined units cannot be formed due to too small amount of measurements. HSC will notify you about that after pressing the calculate button (combined units can't be formed).
  When the estimation of missing is not done before balance calculation, HSC will do the balancing using the raw measurements only and then back calculates the missing values, in this order the results are often more reliable.
- ✓ Estimated values (indicated with blue font) can be cleared away. Typically, the estimation is not needed prior balancing.
- ✓ Create several data to represent several sampling rounds. This can be done by pressing the "Add Round" button. Pressing "Calculate Master Data" will let HSC calculate the master data using the average of the data in the sampling rounds.
- ✓ Set the total solids of a stream to be a multiple of another stream by pressing "Stream by Equation".

51.4.3. Reviewing and complementing the data

In this part, the idea is to inspect the data and get an understanding of what values will be available after balancing. Also, the data status before balancing is reviewed here and can be changed. These changes are reflected in the status after balancing and shown graphically.

The status indications are:

- Stream:
  - Missing: no data
  - Complete: all variables have data
  - Partial: some variables have data

- Variable:
  - Missing: no data
  - Measured: data existing
  - Guesstimated: data are a user-given guesstimation; high uncertainty is set automatically for this The guesstimation is given in **measured column after the status is set guestimated**
  - Fixed: the user sets including the value that you do not wish to change during the balancing. The fixed value is given in the property **measured column after the status is set guestimated**
  - Excluded: data will not be included in the calculation

- After Balancing:
  - Balanced: solved using data reconciliation
  - Calculated: calculated based on unit material balance
  - Non Available/backcalculated: data are also missing after balancing or it will be backcalculated

The status indications can be changed by clicking the data table cell (dropdown menu) or from the property panel on the right.

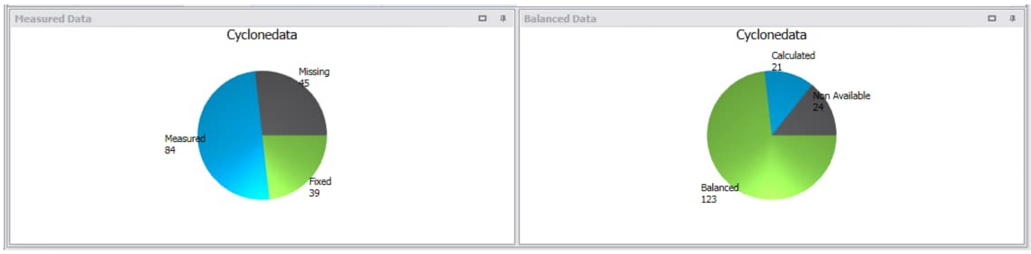


Fig. 10. An example of graphical status indicators.

- ✓ The balance equations for the units can be reviewed by clicking the upper bar button shown.

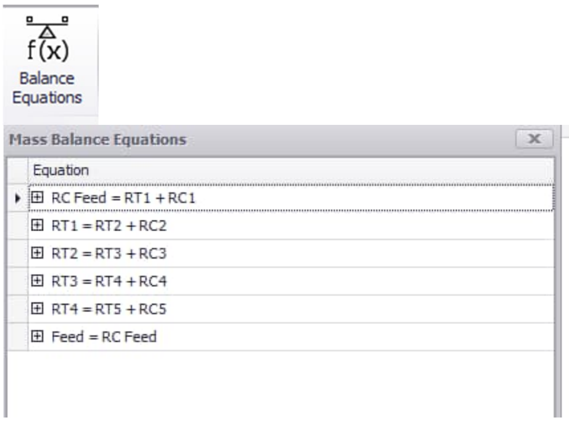


Fig. 11. Opening and reviewing the balance equations.

- ✓ Check for errors will indicate if any problems preventing the balance calculation exists.
- ✓ All streams with no data at all can be easily unselected here by one button click.
- ✓ Balancing of minerals is (automatically) selected if mineral variables with data exists, balance is calculated then using minerals and the elements, if any, are back calculated from minerals automatically
- ✓ All measurement data status changes can be reset
- ✓ The total solids of a stream can be set as a multiple of another stream. After doing this, the corresponding cell color will turn to light green.

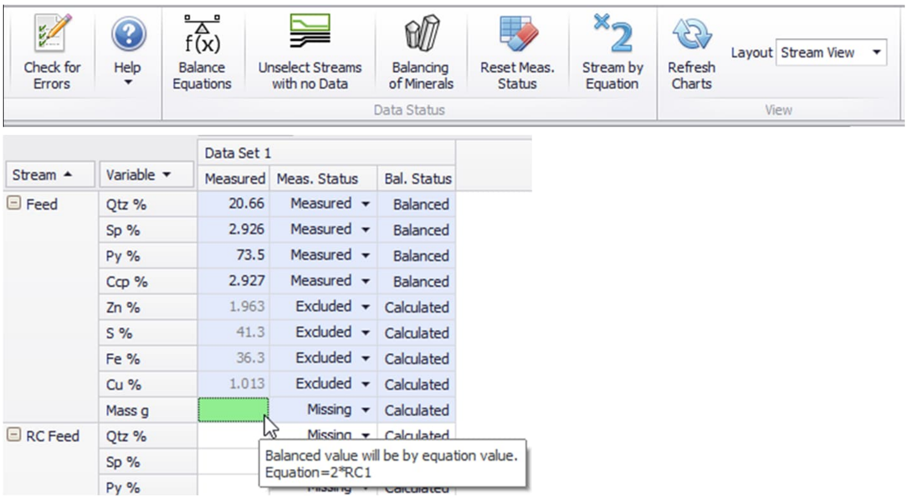


Fig. 12. Data Status View

51.4.4. Setting the measurement accuracies

Each assay and piece of raw data is subject to errors. Mass balancing and data reconciliation is meant for adjusting unreliable values, whereas reliable values should be adjusted only a little, if at all. Therefore, the user has to give a value of how reliable each item of raw data is. This is done by defining the error model to give a standard deviation value for the measurement data.

In the HSC Sim Mass Balance tool, the standard deviation can be given using user-friendly pre-settings for the error models.

To define the standard deviations you need to select:

Table 3. Descriptions of error model functions.

Error Model	**Parameters**	**Example**
Fixed Absolute	Absolute SD value is given in the same engineering unit as the variable. The relative RSD % is then calculated based on it and shown in blue font	a = 5
Fixed Relative	Relative SD % value is given as a percentage. The absolute SD is then calculated based on it and shown in blue font	a = 5
S-shape	This is typical for analytical measurement devices. The SD is calculated for each value by using parameters: relative error = RSD % (a), detection limit = minimum possible SD (b), and error saturation = maximum possible SD (c)	a = 5 b = 0.01 c = 0.5
Mineral Grade Based	Will calculate RSD % based on grade value *x* (of the mineral), *RSD = a*x^(-0.5)*	a = 5
Size Fraction (Whiten)	SD of a size fraction assay x is calculated: SD = 0.1 + x/10, if the size assay x > 10, then SD = 1.0	-

Stream sampling accuracies were given in the ‘Streams’ view.

Note: the sampling error is used only in conjunction with measurement methods that are calculated with a formula (S-shape, Mineral Grade Based, Size Fraction (Whiten)). With ***Fixed Absolute*** *and* ***Fixed Relative*** *measurement methods, the* ***sampling error is not in use*** *and it is not added. Please note: Fixed Relative + stream sampling error can be obtained by using the S-shape measurement method with zero detection limit and very high saturation limit and given parameter a = RSD %.*

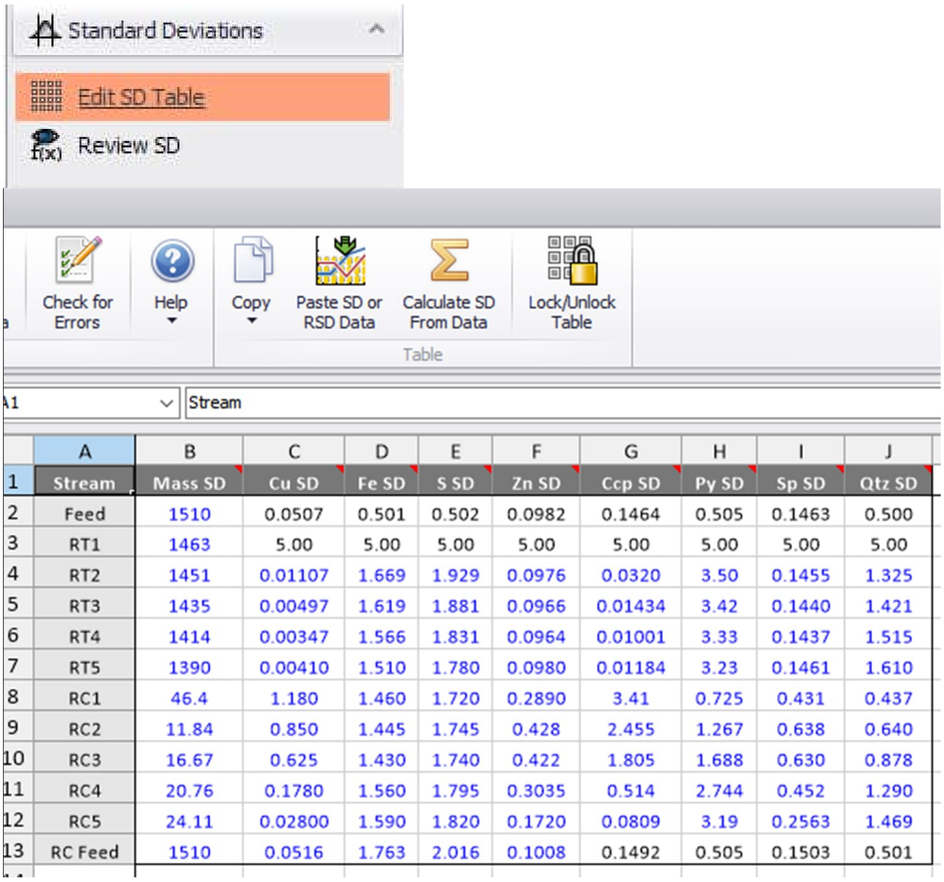
Table 4. Sample quality impact on given sampling error.

Sample Quality	x Sampling Error
Good	1
Moderate	1.5
Bad	3

1. Edit SD view

You can paste and edit **SD** and/or **RSD** values in this view, similarly to importing measurement data earlier. Blue values are indicated as calculated by HSC. In addition, here you can:

- ✓ Calculate the SD based on the data given in the sampling rounds.

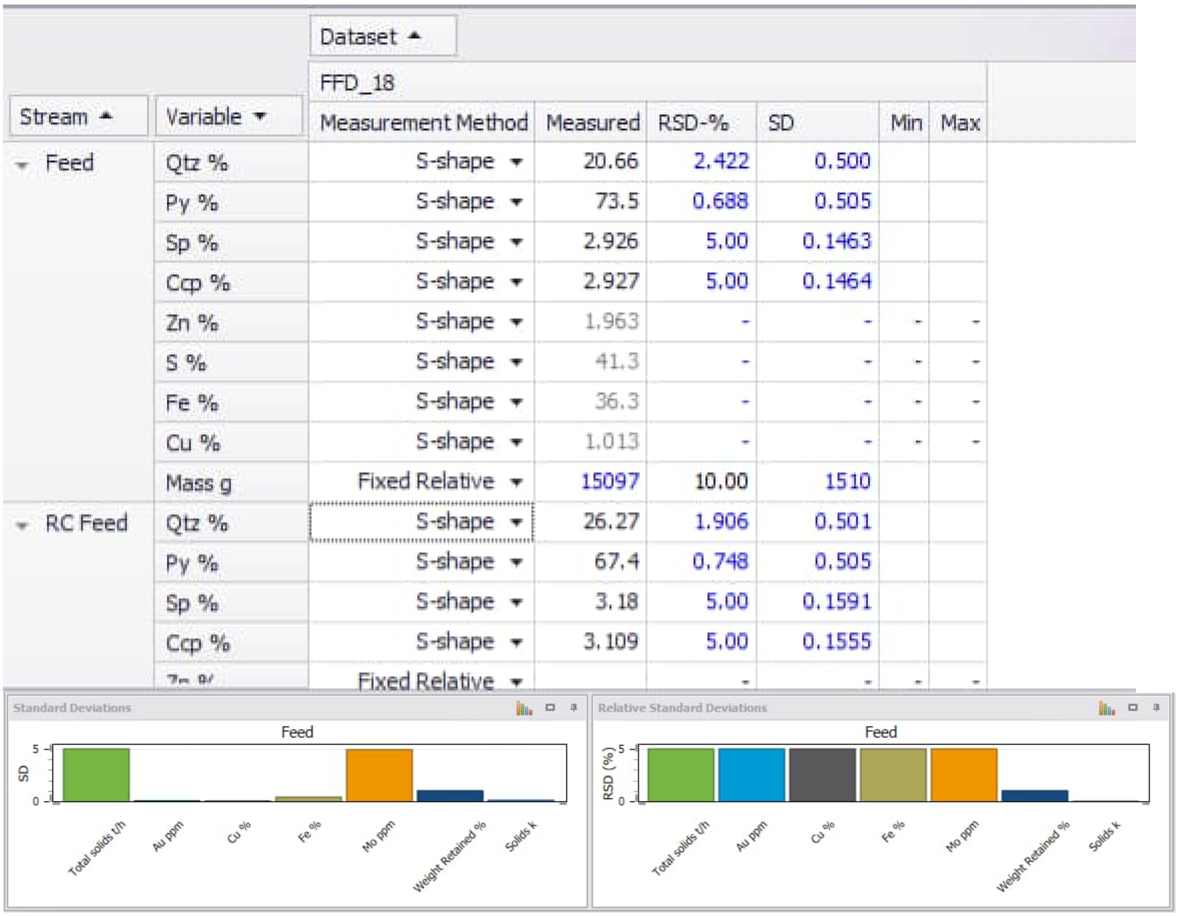


	A	B	C	D	E	F	G	H	I	J
1	Stream	Mass SD	Cu SD	Fe SD	S SD	Zn SD	Ccp SD	Py SD	Sp SD	Qtz SD
2	Feed	1510	0.0507	0.501	0.502	0.0982	0.1464	0.505	0.1463	0.500
3	RT1	1463	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
4	RT2	1451	0.01107	1.669	1.929	0.0976	0.0320	3.50	0.1455	1.325
5	RT3	1435	0.00497	1.619	1.881	0.0966	0.01434	3.42	0.1440	1.421
6	RT4	1414	0.00347	1.566	1.831	0.0964	0.01001	3.33	0.1437	1.515
7	RT5	1390	0.00410	1.510	1.780	0.0980	0.01184	3.23	0.1461	1.610
8	RC1	46.4	1.180	1.460	1.720	0.2890	3.41	0.725	0.431	0.437
9	RC2	11.84	0.850	1.445	1.745	0.428	2.455	1.267	0.638	0.640
10	RC3	16.67	0.625	1.430	1.740	0.422	1.805	1.688	0.630	0.878
11	RC4	20.76	0.1780	1.560	1.795	0.3035	0.514	2.744	0.452	1.290
12	RC5	24.11	0.02800	1.590	1.820	0.1720	0.0809	3.19	0.2563	1.469
13	RC Feed	1510	0.0516	1.763	2.016	0.1008	0.1492	0.505	0.1503	0.501

Fig. 13. Edit SD Table view

2. Review SD

Here, the data and related SD and RSD-% values can be reviewed, and are also presented graphically. You can also change Measurement Methods and their parameters in this view and the property panel



		Dataset					
		FFD_18					
Stream	Variable	Measurement Method	Measured	RSD-%	SD	Min	Max
Feed	Qtz %	S-shape	20.66	2.422	0.500		
	Py %	S-shape	73.5	0.688	0.505		
	Sp %	S-shape	2.926	5.00	0.1463		
	Ccp %	S-shape	2.927	5.00	0.1464		
	Zn %	S-shape	1.963	-	-	-	-
	S %	S-shape	41.3	-	-	-	-
	Fe %	S-shape	36.3	-	-	-	-
	Cu %	S-shape	1.013	-	-	-	-
	Mass g	Fixed Relative	15097	10.00	1510		
RC Feed	Qtz %	S-shape	26.27	1.906	0.501		
	Py %	S-shape	67.4	0.748	0.505		
	Sp %	S-shape	3.18	5.00	0.1591		
	Ccp %	S-shape	3.109	5.00	0.1555		
	Zn %	Fixed Relative		-	-	-	-

Fig. 14. Review SD view.

51.4.5. Balance

In the mass balancing upper bar, the following can be selected:

- Assay Sum = 100: selected when chemical component sum is required to be 100%
- Mineral Sum = 100: selected when the mineral component sum is required to be 100%.
- Method: LS, NNLS, CLS, LLS
- Data to Balance:
  - **Solids only**
  - **Assays only**
  - **Solids and assays**
  - **Solids, water and assays**
- PSD Balance:
  - **Unsized**
  - **Sized**
  - **Sized by Assay**
- Low Grade Weighting

- ✓ **Calculate**: runs the data reconciliation
- ✓ **Freeze balanced**: you can balance only some part of the circuit or some type of balancing (e.g. sized), and then freeze them and fixed. Balancing of the next part of the process can be then continued. This can be repeated many times successively
- ✓ **Clear & Unfreeze Balanced** will unfreeze the results and clear the balanced data table empty. Note: between balancing (without freezing) there is no need to click this button.
- ✓ **Batch run**: runs the selected datasets
- ✓ **Element to Mineral** conversion can be done here also after the elements are balances (mineral based balancing is not necessity even the result are to be presented as minerals)
- ✓ **Show Data Bars** will show the data bar for each variable’s recovery

The balancing results can be viewed graphically with, see **Fig. 15**:

- Balance Convergence
- Parity Chart

To solve a mass balance problem, the following mathematical methods are available in the Balance/Report Options on the right-hand side:

- Least Squares Solution (LS)
- Non-negative Least Squares Solution (NNLS)
- Constrained Least Squares Solution (CLS)
- Limited Least Square (LLS), only available for unsized balancing

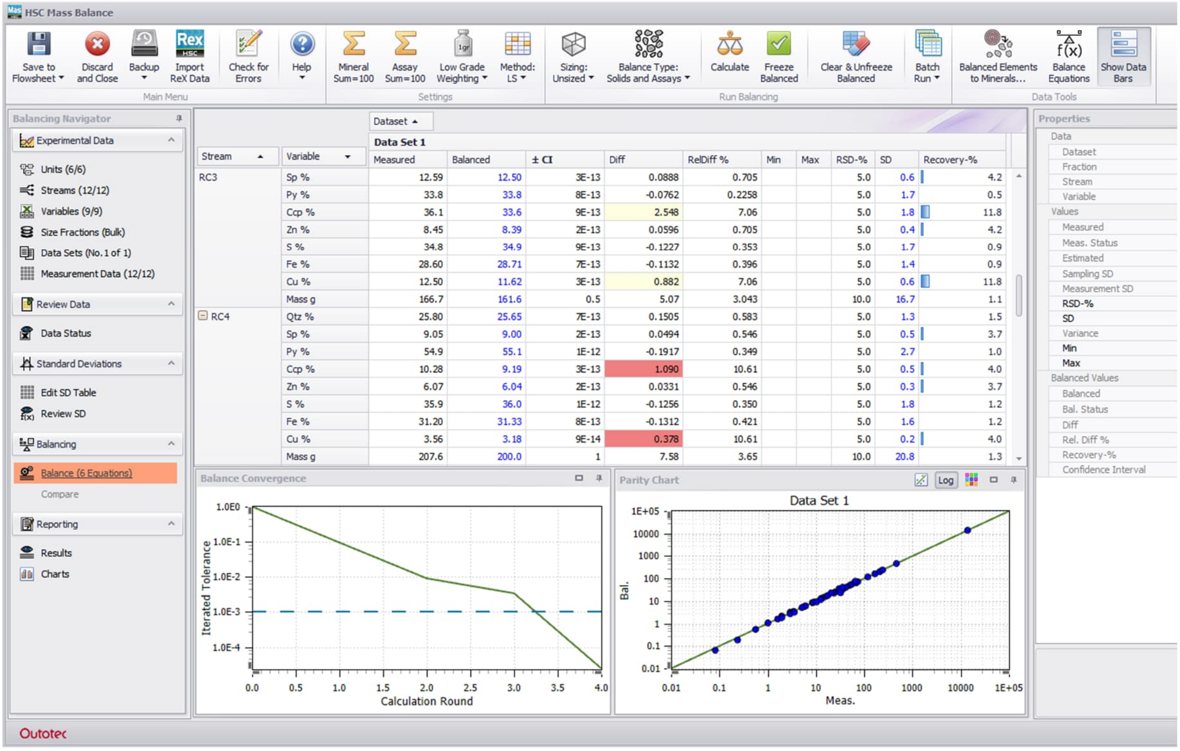


Fig. 15. Balancing – Calculate.

Mass balance problems are solved in two stages: firstly, the total mass flow rates are solved and then the assays are reconciled. In solving the assays, the least squares solution finds the best solution by minimizing the weighted sum of squares, i.e.

$$WSSQ = \sum_{j=1}^{k}\sum_{i=1}^{n}\frac{(a_{ij} - b_{ij})^2}{s_{ij}^2} \tag{1}$$

where j refers to the stream, k is the number of streams, i refers to the components (analyses), n is the number of components, *a* is the measured value, *b* is the balanced value, and s is the standard deviation.

In non-negative least squares, all ‘*b*’s are subject to being non-negative.

In constrained least squares, all ‘*b*’s are subject to being between the min. and max.

In limited least squares, all ‘*b*’s are subject to be greater than the given standard deviation

By clicking dataset you can see the solution parameters Balance tolerance, Max iter and Estimate of null SD in the properties window. Balance tolerance is the condition that defines when the iterations stop and Max iter is the maximum number of iterations. If you don’t get reasonable balance you can try to change Estimate of null SD greater.

Scaling:

By default the scaling parameter in the Balance view right side property panel is selected. This normalises assay and/or fraction% data. This will typically result more reliable and robust balance calculation. If the flow rate balance for some reason fails (most of them are around zero), try to uncheck the scaling option.

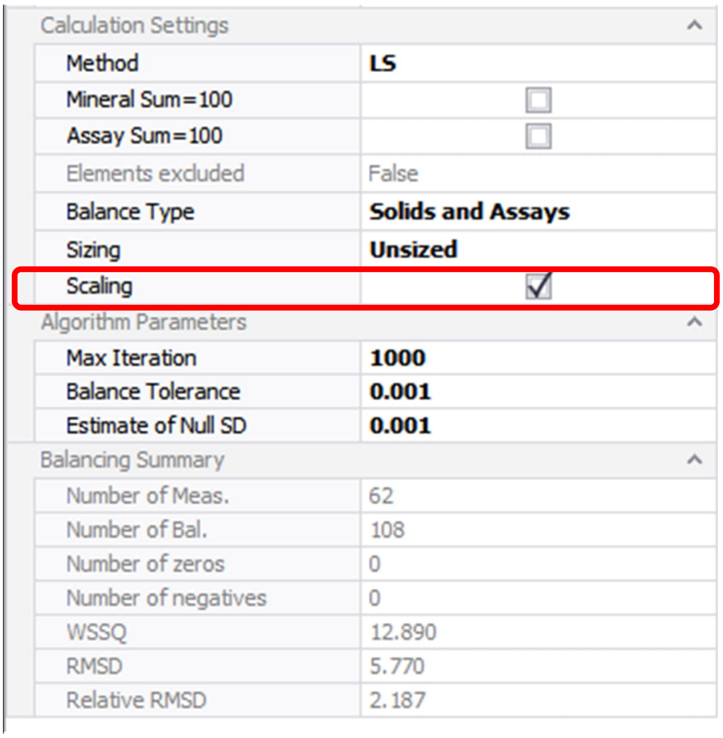


Fig. 16. Property panel, scaling of data prior balancing.

The assay measurements and/or fraction% measurements are normalised by using their *feed stream* SD values when solving the flow rates with *EWTLS* method. This will reinforce the impact of both assays and fraction% in the flow rate balance calculation.

Scaling is done as follows (Matlab notation):

```
% Scaling of assay measurements (8 streams, 13 assays):
indexOfInput = 2;
S = repmat(AssaysSD(indexOfInput, :), size(AssaysSD, 1), 1);
Assays_scaled = Assays ./ (100 * S);
```

Fig. 17. Scaling of assays in Matlab.

Parameter **`indexOfInput`** refers to the feed stream which in this example corresponds to the second row in two input arrays (**`Assays`** and **`AssaysSD)`**.

```
Assays =

    1.3360    4.9610    0.0017    0.0722   25.1150   28.8150    5.2430    0.0040    2.2500
    4.1833    2.8075    0.0009    0.1215   12.8402   14.4975    4.8978    0.0142    3.5225
    2.0943    4.2838    0.0015    0.0777   21.6077   25.6525    5.0287    0.0064    2.7350
    1.8275    4.1692    0.0015    0.0712   19.9477   22.4175    4.9335    0.0058    2.9550
    1.6087    2.6412    0.0009    0.0680   11.6930   13.4150    4.3422    0.0058    4.0125
    2.1522    4.9405    0.0017    0.0833   25.2105   28.5450    5.2287    0.0067    2.3450
   41.0950    3.4228    0.0011    0.4642   13.5017   22.8225    3.4295    0.1359    1.7625
    2.3763    3.4522    0.0011    0.0850   14.0290   16.8525    4.8805    0.0076    3.6475

AssaysSD =

    0.1282    0.1488    0.0002    0.0043    1.0534    1.3874    0.1302    0.0005    0.2218
    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240
    0.2437    0.0924    0.0001    0.0095    0.7606    1.2515    0.2379    0.0008    0.0332
    0.3956    0.5286    0.0002    0.0079    2.7793    3.5568    0.2428    0.0008    0.5089
    0.1404    0.1289    0.0001    0.0104    0.5021    0.9676    0.1973    0.0008    0.2791
    0.0703    0.1861    0.0001    0.0071    0.4044    0.7750    0.1814    0.0003    0.1595
    5.5495    0.9885    0.0001    0.0626    2.5093    1.5387    0.5361    0.0208    0.0359
    1.1302    0.2125    0.0001    0.0155    1.1902    1.9589    0.1675    0.0030    0.3186

S =

    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240
    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240
    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240
    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240
    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240
    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240
    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240
    0.3684    0.3553    0.0001    0.0418    1.0808    2.8585    0.6189    0.0033    0.4240

Assays_scaled =

    0.0363    0.1396    0.1689    0.0173    0.2324    0.1008    0.0847    0.0121    0.0531
    0.1136    0.0790    0.0892    0.0291    0.1188    0.0507    0.0791    0.0425    0.0831
    0.0568    0.1206    0.1445    0.0186    0.1999    0.0897    0.0812    0.0193    0.0645
    0.0496    0.1173    0.1425    0.0171    0.1846    0.0784    0.0797    0.0174    0.0697
    0.0437    0.0743    0.0826    0.0163    0.1082    0.0469    0.0702    0.0175    0.0946
    0.0584    0.1390    0.1647    0.0199    0.2333    0.0999    0.0845    0.0201    0.0553
    1.1155    0.0963    0.1087    0.1111    0.1249    0.0798    0.0554    0.4081    0.0416
    0.0645    0.0972    0.1025    0.0203    0.1298    0.0590    0.0789    0.0230    0.0860
```

Figure 18. Scaling -- A numerical example (only nine data columns are shown).

Low Grade Weighting:

To enable low grade weighting for selected components (e.g. Au), click *the Low Grade Weighting* button in the *Balance* view and select the component(s), **Fig. 19**. The weighting coefficient should be selected as follows:

- Coefficient = 0, no weighting, same as if this were not selected at all
- Coefficient = 1 … 1.5, heavy weighting, larger values than 1.5 should not be used
- A good starting point for selecting the weighting is **0.7**

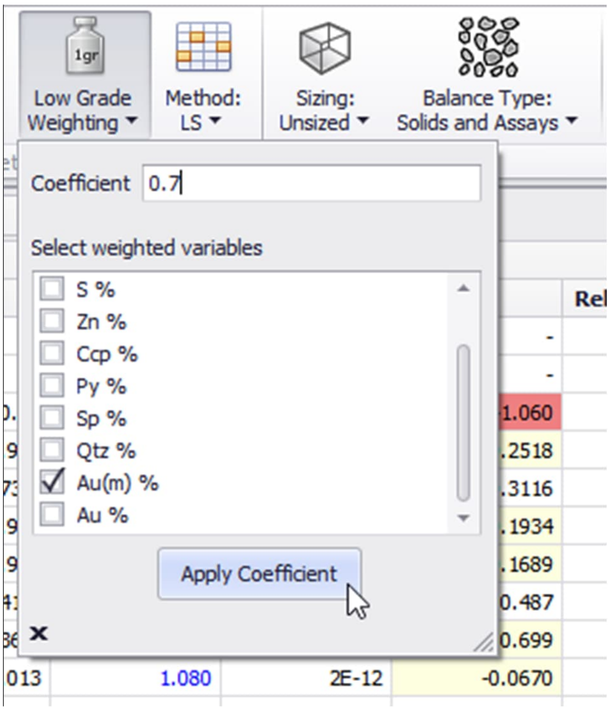


Fig. 19. Selecting of low grade weighting.

51.4.6. Reporting and reviewing the results

Here, you can get the mass balancing results in both table and chart form.

A1 Stream

	A	B	C	D	E	F	G	H	I
1	Stream	Mass g Meas.	Mass g Bal.	Mass Rec %	Cu % Meas.	Cu % Bal.	Cu Rec %	Fe % Meas.	Fe % B
2	Feed		***15097***	100.0	1.013	1.124	100.0	36.3	30.2
3	RT1		***14633***	96.9		0.413	35.6		30.3
4	RT2		***14514***	96.1		0.2775	23.72		30.3
5	RT3		***14348***	95.0		0.1358	11.47		30.4
6	RT4		***14140***	93.7		0.0862	7.18		30.4
7	RT5	13899	***13899***	92.1	0.0820	0.0780	6.39	30.20	30.4
8	RC1	464	***464***	3.075	23.60	23.56	64.4	29.20	27.6
9	RC2	118.4	***118.4***	0.784	17.00	16.99	11.85	28.90	26.8
10	RC3	166.7	***166.7***	1.104	12.50	12.47	12.25	28.60	26.8
11	RC4	207.6	***207.6***	1.375	3.56	3.51	4.30	31.20	28.8
12	RC5	241.1	***241.1***	1.597	0.560	0.556	0.789	31.8	30.4
13	RC Feed		***15097***	100.0		1.124	100.0		30.2
14									

Fig. 20. Stream Summary

A1 Variable

	A	B	C	D	E	F	G
1	Variable	WSSQ	Diff.Tot	Rel.Dif	AVG SD	AVG RSD	RMSD
2	Mass g	0.000362	0.00369	3.58E-06	733	0.0855	3.77E-04
3	Cu %	65.4	0.643	1.066	0.1710	0.0342	0.065
4	Fe %	15.33	36.6	10.37	1.699	0.0524	3.51
5	S %	12.10	25.81	6.02	1.912	0.0508	2.98
6	Zn %	4.24	1.639	3.50	0.2225	0.0585	0.17
7	Ccp %	4.00	1.434	0.823	0.2350	0.01625	0.144
8	Sp %	4.61	1.849	2.653	0.2529	0.0442	0.19
9	Py %	30.60	37.0	5.74	0.468	0.00831	4.34
10	Qtz %	29.33	33.7	10.83	0.459	0.01956	4.02
11							
12	Stream	Sum WSSQ	Diff.Sum	Rel.Diff.Sum	Rel.Diff.Avg	RMSD	Relative RMSD
13	Feed	28.97	21.19	0.1394	0.0994	6.47	129.3
14	RT1	12.90	9.41	0.0639	0.0994	2.870	57.4
15	RT2	13.02	9.49	0.0649	0.0994	2.894	57.9
16	RT3	13.18	9.60	0.0665	0.0994	2.930	58.6
17	RT4	13.40	9.76	0.0685	0.0994	2.976	59.5
18	RT5	12.03	1.187	0.00848	0.001005	0.365	7.29
19	RC1	1.265	1.025	0.1816	0.00453	0.2721	5.44
20	RC2	0.450	0.376	0.1721	0.01153	0.1007	2.013
21	RC3	0.797	0.670	0.2513	0.00949	0.1785	3.57
22	RC4	1.202	1.012	0.329	0.00812	0.2695	5.39
23	RC5	1.554	1.162	0.341	0.00562	0.344	6.89
24	RC Feed	12.43	9.10	0.0599	0.0994	2.778	55.6

Fig. 21. Reporting – Results: Goodness.

A1 Units

Units	Equations	Streams	Mass g	Cu g	Fe g	S g	Zn g	
Rougher 1	RC Feed = RT1 + RC1							
	Inputs	RC Feed	15097	169.8	4569	5414	342	
	Outputs	RT1	14633	60.4	4440	5254	315.1	
		RC1	464	109.4	128.6	160.6	26.46	
	Balance		1.76E-11	-2.40E-12	1.79E-12	1.96E-12	5.12E-13	-6
	Tolerance Error		1.17E-15	1.41E-14	3.92E-16	3.62E-16	1.50E-15	1
Rougher 2	RT1 = RT2 + RC2							
	Inputs	RT1	14633	60.4	4440	5254	315.1	
	Outputs	RT2	14514	40.3	4408	5212	305.0	
		RC2	118.4	20.11	31.7	41.4	10.10	
	Balance		2.00E-11	-6.57E-13	-1.18E-12	5.83E-13	6.85E-12	-1
	Tolerance Error		1.37E-15	1.09E-14	2.65E-16	1.11E-16	2.18E-14	1
Rougher 3	RT2 = RT3 + RC3							
	Inputs	RT2	14514	40.3	4408	5212	305.0	
	Outputs	RT3	14348	19.48	4364	5154	291.0	
		RC3	166.7	20.79	44.7	58.2	14.01	
	Balance		2.71E-11	-7.46E-14	-5.54E-13	4.69E-13	4.65E-13	-2
	Tolerance Error		1.87E-15	1.85E-15	1.26E-16	9.00E-17	1.53E-15	1
Rougher 4	RT3 = RT4 + RC4							
	Inputs	RT3	14348	19.48	4364	5154	291.0	
	Outputs	RT4	14140	12.18	4304	5079	278.5	
		RC4	207.6	7.29	59.9	74.9	12.48	
	Balance		3.72E-11	-3.61E-13	-3.47E-12	-4.23E-12	-1.66E-12	-1

Fig. 22. Reporting – Unit Balance.

A1 Stream

Stream	Mass g Meas.	Mass g Bal.	Mass g±CI	Mass g SD	Mass g RSD	Mass g Diff	Mass g Rel.Diff %	Cu % Me
Feed		15097	1	1510	10.00			1.013
RT1		14633	0.9	1463	10.00			
RT2		14514	0.9	1451	10.00			
RT3		14348	1	1435	10.00			
RT4		14140	1	1414	10.00			
RT5	13899	13899	2	13.90	0.1000	-0.000594	4.28E-06	0.082
RC1	464	464	0.4	0.682	0.1470	-2.27E-05	4.88E-06	23.60
RC2	118.4	118.4	0.5	0.514	0.434	-2.09E-05	1.77E-05	17.00
RC3	166.7	166.7	0.7	0.527	0.3162	-2.47E-05	1.48E-05	12.50
RC4	207.6	207.6	1	0.541	0.2608	-3.57E-05	1.72E-05	3.56
RC5	241.1	241.1	2	0.555	0.2302	0.0001924	7.98E-05	0.560
RC Feed		15097	1	1510	10.00			

Fig. 23. Reporting_Balance details.

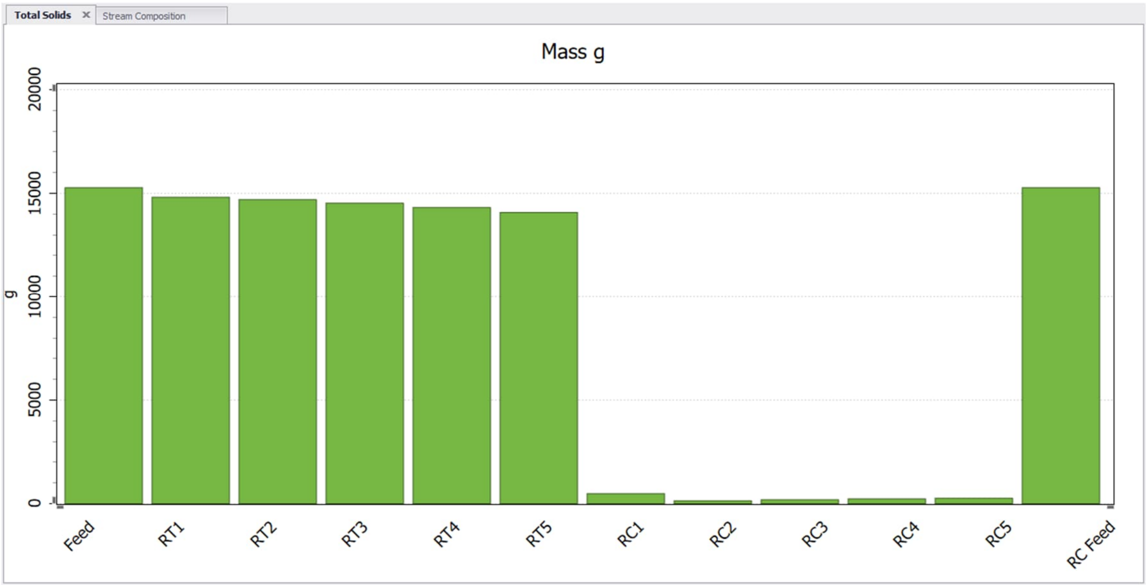


Fig. 22. Reporting_Charts_Total Solids

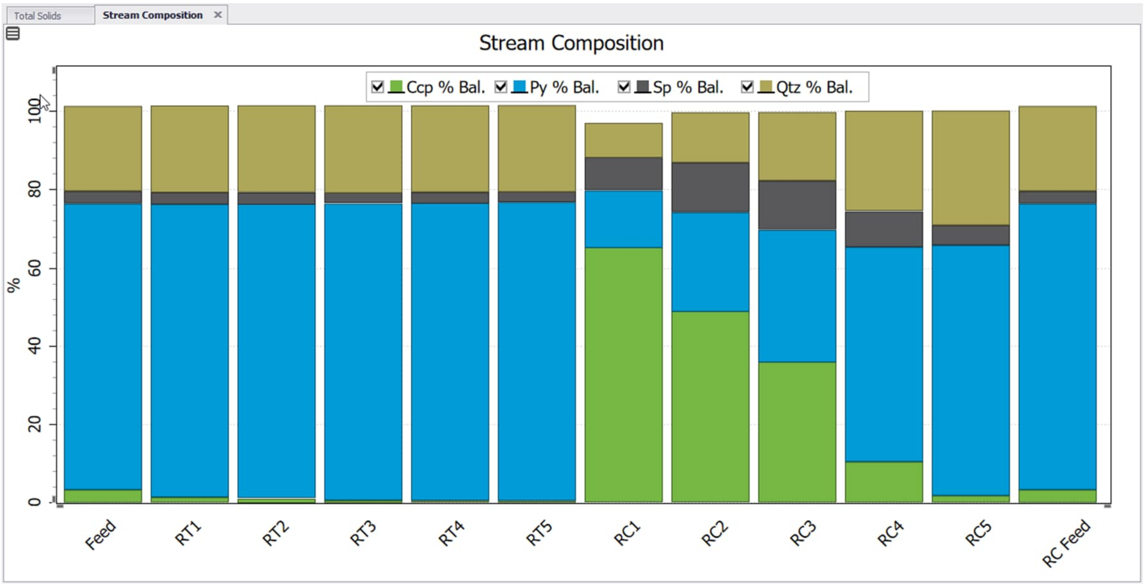
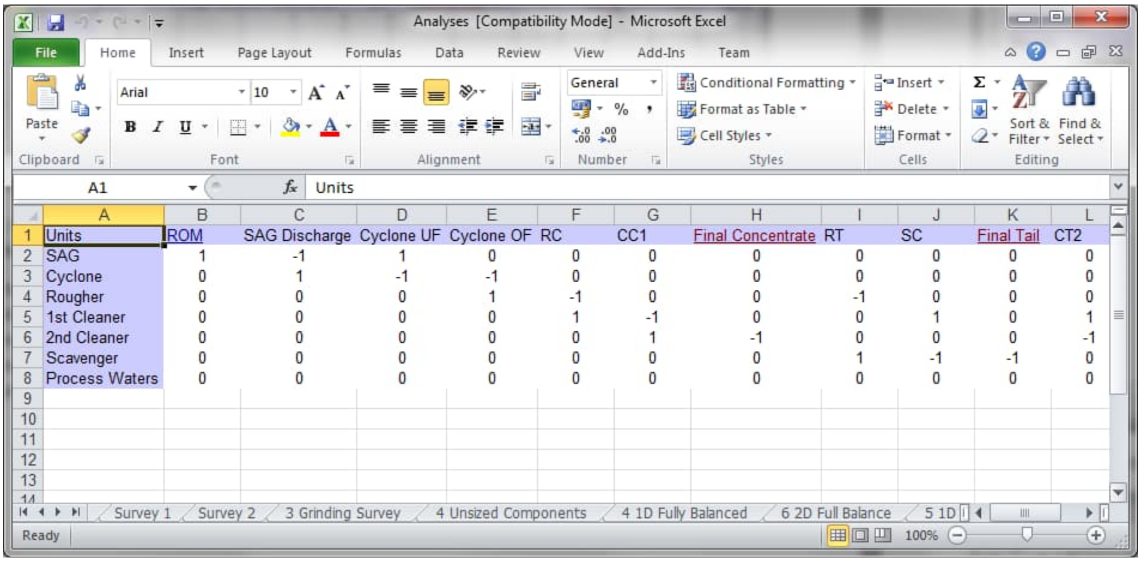


Fig. 23. Reporting_Charts_Stream Composition

51.4.7. Importing HSC7 Excel files

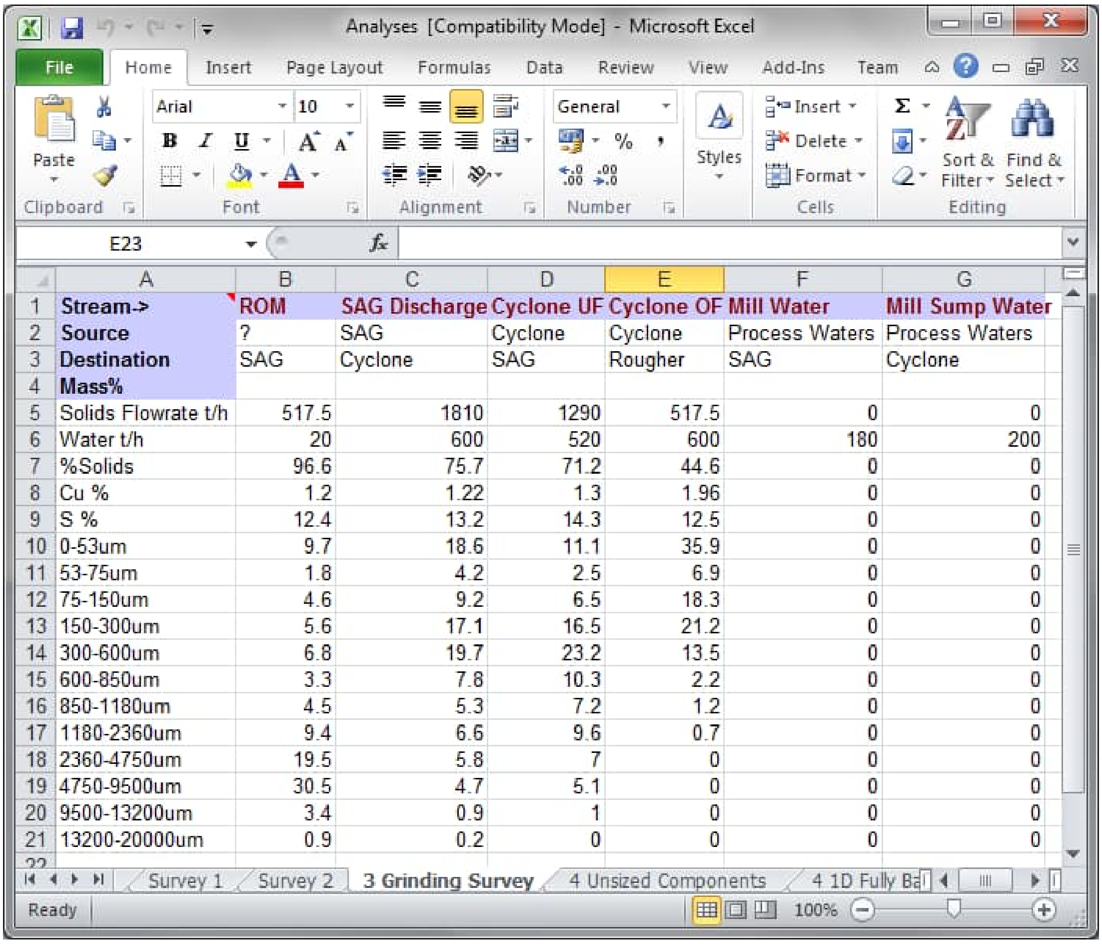
The Excel files to be imported may contain a sheet with flowsheet information (**Fig. 24**). If this sheet is named Flowsheet, the program automatically detects the sheet where the flowsheet information is located. If the name is something else or there is no sheet containing flowsheet information, the name must be specified (**Fig. 28**, Select Flowsheet)



Units	ROM	SAG Discharge	Cyclone UF	Cyclone OF	RC	CC1	Final Concentrate	RT	SC	Final Tail	CT2
SAG	1	-1	1	0	0	0	0	0	0	0	0
Cyclone	0	1	-1	-1	0	0	0	0	0	0	0
Rougher	0	0	0	1	-1	0	0	-1	0	0	0
1st Cleaner	0	0	0	0	1	-1	0	0	1	0	1
2nd Cleaner	0	0	0	0	0	1	-1	0	0	0	-1
Scavenger	0	0	0	0	0	0	0	1	-1	-1	0
Process Waters	0	0	0	0	0	0	0	0	0	0	0

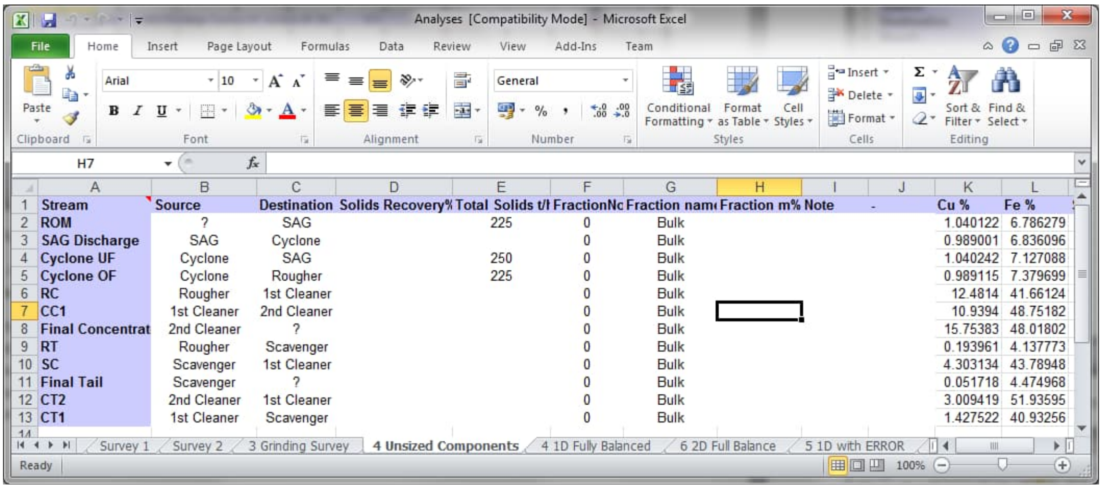
Fig. 24. An Excel sheet containing flowsheet data.

The measurement data is read from a different sheet of the same Excel file. The first column must be named Streams (Cell A1), the second Source (Cell A2), and the third Destination (cell A3) if the data are horizontal (**Fig. 26**). If the data are vertical, the first row must be named Streams (A1), the second row must be named Source (A2), and the third Destination (A3) (**Fig. 25**). Imported vertical data can only be unsized or sized (without analyses).



	A	B	C	D	E	F	G
1	Stream->	ROM	SAG Discharge	Cyclone UF	Cyclone OF	Mill Water	Mill Sump Water
2	Source	?	SAG	Cyclone	Cyclone	Process Waters	Process Waters
3	Destination	SAG	Cyclone	SAG	Rougher	SAG	Cyclone
4	Mass%						
5	Solids Flowrate t/h	517.5	1810	1290	517.5	0	0
6	Water t/h	20	600	520	600	180	200
7	%Solids	96.6	75.7	71.2	44.6	0	0
8	Cu %	1.2	1.22	1.3	1.96	0	0
9	S %	12.4	13.2	14.3	12.5	0	0
10	0-53um	9.7	18.6	11.1	35.9	0	0
11	53-75um	1.8	4.2	2.5	6.9	0	0
12	75-150um	4.6	9.2	6.5	18.3	0	0
13	150-300um	5.6	17.1	16.5	21.2	0	0
14	300-600um	6.8	19.7	23.2	13.5	0	0
15	600-850um	3.3	7.8	10.3	2.2	0	0
16	850-1180um	4.5	5.3	7.2	1.2	0	0
17	1180-2360um	9.4	6.6	9.6	0.7	0	0
18	2360-4750um	19.5	5.8	7	0	0	0
19	4750-9500um	30.5	4.7	5.1	0	0	0
20	9500-13200um	3.4	0.9	1	0	0	0
21	13200-20000um	0.9	0.2	0	0	0	0

Fig. 25. Vertical sized data (without analyses).



	A	B	C	D	E	F	G	H	I	J	K	L
1	Stream	Source	Destination	Solids Recovery%	Total Solids t/I	FractionNc	Fraction nam	Fraction m%	Note	-	Cu %	Fe %
2	ROM	?	SAG		225	0	Bulk				1.040122	6.786279
3	SAG Discharge	SAG	Cyclone			0	Bulk				0.989001	6.836096
4	Cyclone UF	Cyclone	SAG		250	0	Bulk				1.040242	7.127088
5	Cyclone OF	Cyclone	Rougher		225	0	Bulk				0.989115	7.379699
6	RC	Rougher	1st Cleaner			0	Bulk				12.4814	41.66124
7	CC1	1st Cleaner	2nd Cleaner			0	Bulk				10.9394	48.75182
8	Final Concentrat	2nd Cleaner	?			0	Bulk				15.75383	48.01802
9	RT	Rougher	Scavenger			0	Bulk				0.193961	4.137773
10	SC	Scavenger	1st Cleaner			0	Bulk				4.303134	43.78948
11	Final Tail	Scavenger	?			0	Bulk				0.051718	4.474968
12	CT2	2nd Cleaner	1st Cleaner			0	Bulk				3.009419	51.93595
13	CT1	1st Cleaner	Scavenger			0	Bulk				1.427522	40.93256

Fig. 26. Horizontal unsized data.

The program automatically detects a sheet named Streams as the sheet where the measurement data can be found. If the name of the sheet containing the measurement data is not Streams, the user should tell the program where the measurement data can be found (**Fig. 28**, Analyses)

The following screenshots show how an HSC7 file is imported. Importing an HSC7 file is started by clicking Tools-Mass Balancing and then clicking the Import HSC7 Data button (**Fig. 27**). After that a dialog will open (**Fig. 28**).

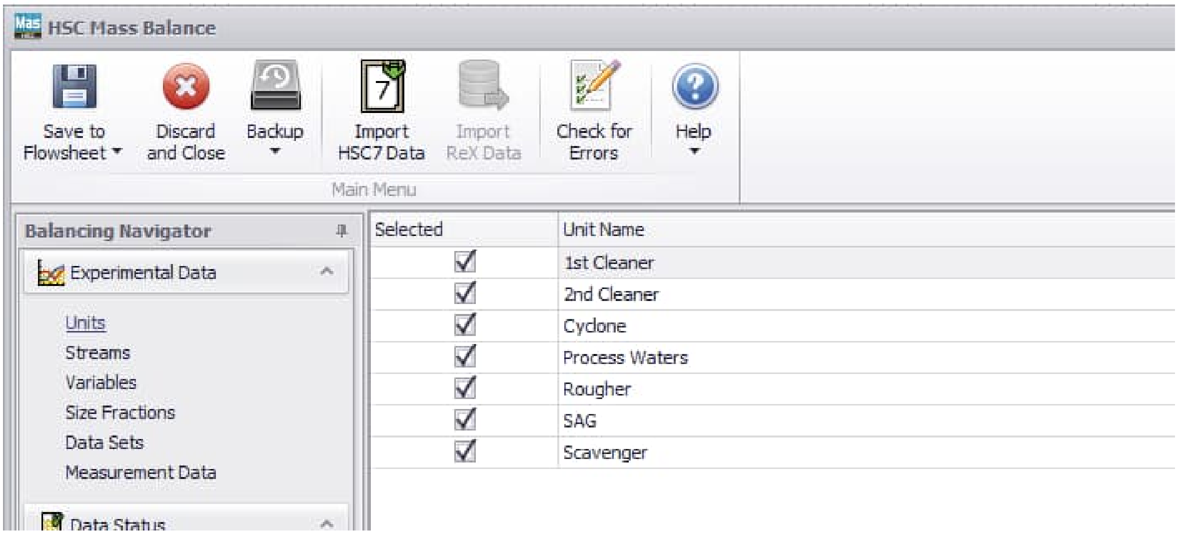


Fig. 27. Import HSC7 Data.

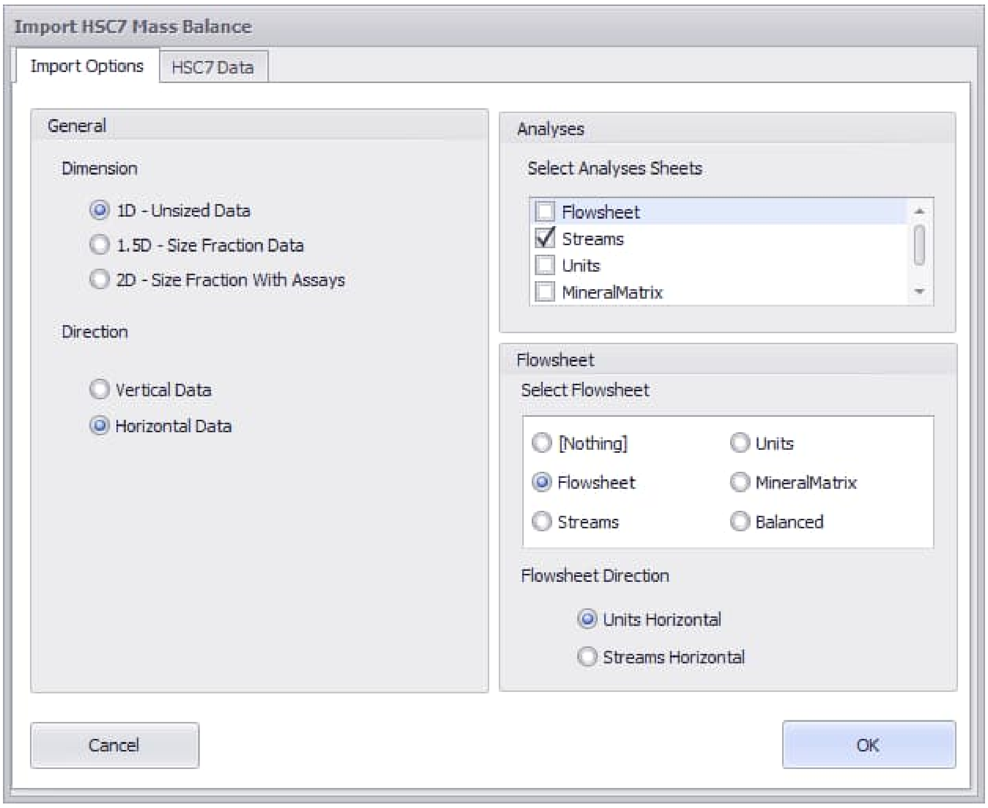
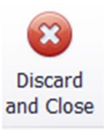


Fig. 28. Import HSC7 Data Dialog.

51.5. Mass balance buttons and dropdowns

Save changes to HSC Sim flowsheet

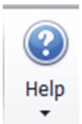


Discard changes and close HSC mass balance

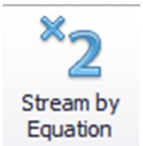
Open/Save backup (HSCMas file) of the mass balance

Import existing HSC7 mass balance data (analyses.xls)

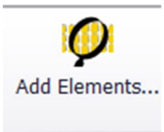
Error check



Help and examples

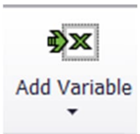


Set the total solid of a stream to be a multiple of other stream

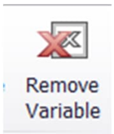


Select elements from periodic table

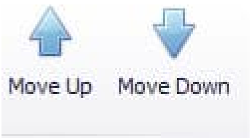
Select minerals from HSC database



Add new variable



Remove the selected variable



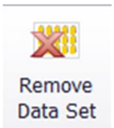
Move Up ,Move Down

Import the minerals from HSC stream files (*.HSCStream)

Add a new sieve size

Remove Size Fraction

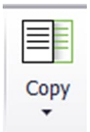
Add a new data set



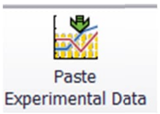
Remove the selected data set

Clones the dataset

Transpose streams and variables



Copy the whole table, template or selection

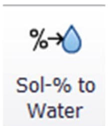


Paste data to correct places based on the stream and variable names

Locks table from editing

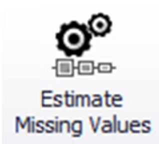
Element to mineral conversion using HSC Geo
For element to mineral conversion, please refer to *HSC Geo* user manual, chapter 84.6

View minerals and their compositions



Converts solids percentage measurements to water flow rates

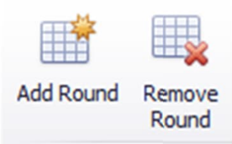
Calculate and show the specific gravity for each stream



Estimate missing data

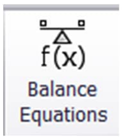
Clear estimated data

Enter recoveries from model fit

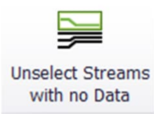


Add and remove sampling rounds

Calculate the master data based on the data in the sampling rounds



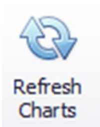
View the mass balance equations



Unselect the streams without data

Balancing of minerals. Solids component assays are excluded if pressed down.

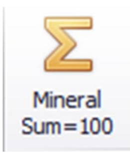
Reset all changes made to measurement status



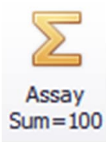
Refresh the charts to reflect the current measurement status

Paste SD or RSD data from the clipboard.

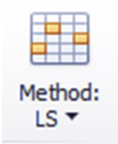
Calculate SD from the data in the sampling rounds



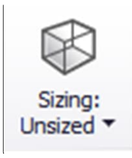
Set the mineral component sum = 100 when balancing



Set the chemical component sum = 100 when balancing



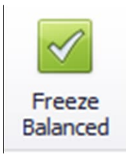
Select calculation method. The available methods are LS, NNLS, CLS, and LLS



PSD balance: Possible selections are Unsized, Sized and Sized by Assays

Select the data to be balanced

Show data bars for the recovery of each stream



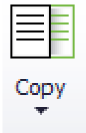
Freeze balanced values

Apply low grade weighting

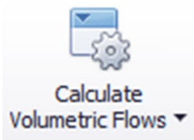
Calculates mass balance

Run selected datasets

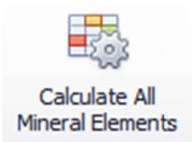
Clears and Unfreezes the mass balancing results



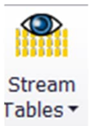
Copy selection
Copy all



Calculate the volumetric flow for each stream using the specified liquid SG

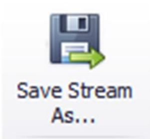


Calculate all the elements of the minerals in the system (not just the elements that were given earlier as variables)

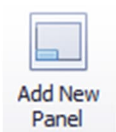


Create or update HSC Sim stream tables

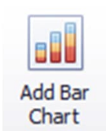
Export the result tables to MS Excel



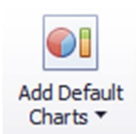
Save the selected stream as HSC Stream files



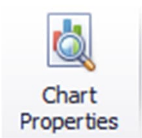
Add new panel / tab in the chart reporting



Add new bar chart to the current panel



Add a default chart (Total Solids or Stream Composition) to the current panel



View and edit the selected chart properties

Reset all changes made to the charts

51.6. Error check messages

1. Units: No selected units found
   - At least one unit should be selected
2. Streams: No selected streams found
   - Some of the streams should be selected
3. Units: No input stream
   - Found a unit with no input streams
4. Units: No output stream
   - Found a unit with no output streams
5. Streams: Source and destination missing
   - Found a stream with no source and destination
6. Data: No data found
   - Add data in the measurement data view
7. Variables: No selected assays found
   - There should be at least one selected variable of the Mineral Assay or Solids Component Assay type
8. Streams: All stream types unknown
   - Go to Streams and press Detect Stream Types button or set the stream types manually

.9 Stream Types: Unknown found
   - Go to Streams and press Detect Stream Types button or set the stream types manually

10. Units: No combined units found
    - There can be several reasons why combined units cannot be formed. There may be too many missing measurements or there may be errors in the flowsheet.
11. Notification: Min or Max values detected: It is recommended to use the CLS method

12. Notification: Method LLS not available for sized solution: It is recommended to use LS, NNLS, or CLS method

51.7. Mathematics and algorithms

In this section, the main algorithms and methods for data reconciliation in HSC Chemistry are briefly summarized.

Definitions

N_{F1}	=	number of flows, unsized
N_{F2}	=	number of flows, sized, sized by assay
N_U	=	number of units
N_{SRU}	=	number of size-reducing units (PSD changing unit)
N_{SF}	=	number of sub-flows
N_E	=	number of chemical elements
G	=	grade of chemical element
G^M	=	measured grade of chemical element
F	=	solids flow rate
F^M	=	measured solids flow rate
W	=	water flow rate
W^M	=	measured water flow rate
Pc	=	%solids = F*100/(F + W)
Pc^M	=	measured %solids
M	=	fraction m% = $F_{flow,subflow}$*100/F_{flow}
M^M	=	fraction m% measurements

$$e_{flow}^{unit} = \begin{cases} 1, \text{the flow enters the unit} \\ -1, \text{the flow exits the unit} \\ 0, \text{otherwise} \end{cases}$$

51.7.1. Unsized (bulk) mass balance

In the *unsized (bulk)* mass balance solution, solids and water flow rates are solved first. After that the bulk analyses are solved. During the calculation: 1) the solids flow rates are solved first and after that 2) the water flow rates are solved.

1. Bulk flow rates

The equations for solving the bulk solids flow rate are:

mass balance Equations (2)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} \cdot F_{flow,tot} = 0 \quad \text{unit} = 1,\ldots,N_U \tag{2}$$

analyses Equation (3)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} \cdot G_{flow,tot,chemical\_element}^{M} \cdot F_{flow,tot} \approx 0 \quad \text{unit} = 1,\ldots,N_U;\ \text{chemical\_element} = 1,\ldots,N_E \tag{3}$$

solids flow rate measurements (4)

$$F_{flow,tot} \approx F_{flow,tot}^{M} \quad \text{flow} = 1,\ldots N_{F1} \tag{4}$$

The equations for solving the bulk water flow rate are:

mass balance Equations (5)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} \cdot W_{flow,tot} = 0 \quad \text{unit} = 1,\ldots,N_U \tag{5}$$

water flow rate measurements Equation (6)

$$W_{flow,tot} \approx W_{flow,tot}^{M} \quad \text{flow} = 1,\ldots N_{F1} \tag{6}$$

% solids measurements (7)

$$(Pc^M/100) \cdot W_{flow,tot} \approx F_{flow,tot} - (Pc^M/100) \cdot F_{flow,tot} \quad \text{flow} = 1,\ldots,N_{F1} \tag{7}$$

Mass balance Equations (2) and (5) are the equality constraints for the solutions. The solution method used is element-wise weighted total least squares[2]. The weights are standard deviations of the solids flow rate measurements F^M, the analyses G^M, the water flow rate measurements W^M, and the %solids measurements Pc^M. The flow rates can be solved without any constraints (LS), subject to non-negativity constraints (NNLS)[3], subject to simple bounds

$$lb1 \le F \le ub1$$
$$lb2 \le W \le ub2$$

(CLS)[4], and subject to be greater than the standard deviation (LLS).

$$F > SD_F$$
$$W > SD_W$$

2. Bulk analyses

Let G and G_M be N_{F1} x N_E matrices. The operator *vec* stacks the matrix columns into a vector.

The equations for the analyses solution are:

measurements (8)

$$vec(G) \approx vec(G^M) \tag{8}$$

mass balance Equations (9) for the analyses:

$$vec(B * G) = 0 \tag{9}$$

where the N_U x N_{F1} matrix B is defined:

$$B_{unit,flow} = e_{unit}^{flow} \cdot F_{flow,tot} \tag{10}$$

If the option minerals sum=100 is selected, the following Equations (11) are included:

$$\sum_{chemical\ element=1}^{N_E} G_{flow,tot,chemical\_element} = 100 \quad \text{flow} = 1,\ldots,N_{F1} \tag{11}$$

Equations (9) and (11) are the equality constraints for the solution. The solution method used is weighted least squares 1). If Equations (11) are included in Equations (9), the matrix B is $(N_U - 1)$ x N_{F1} to avoid linear dependency of equality constraints.

As before, the analyses can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), subject to simple bounds (CLS), and subject to be greater than the standard deviation (LLS). If there are no constraints, the minimal maximum norm solution can be calculated (LS, MinMax)[5].

51.7.2. Sized mass balance (without sized analyses)

Sized mass balance differs from *unsized (bulk)* solution in that the fraction m% is solved and the fraction m% measurement is used in the solution of total flows. *Sized* differs from *Size by Assay* in that the analyses are not given and the number of flows is the same as in the *unsized* case.

1. Total flow mass balances of the streams

Firstly, the total flows are solved. The Equations are:

mass balance Equations (12)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} \cdot F_{flow,tot} = 0 \quad \text{unit} = 1,\ldots,N_U \tag{12}$$

fraction m% measurements (13)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} \cdot M_{flow,subflow}^{M} \cdot F_{flow,tot} \approx 0 \quad \text{unit} = 1,\ldots,N_U - N_{SRU} - 1; \text{subflow} = 1,\ldots,N_{SF} \tag{13}$$

Units are indexed up to N_U - N_{SRU} - 1 to avoid linear dependency of equality constraints.

Flow measurements (14)

$$F_{flow,tot} \approx F^{M}_{flow,tot} \qquad \text{flow} = 1,\ldots N_{F1} \tag{14}$$

If 1D analyses are given, the following Equations are included:

analyses (15)

$$\sum_{flow=1}^{N_{F1}} e^{unit}_{flow} \cdot G^{M}_{flow,tot,chemical\_element} \cdot F_{flow,tot} \approx 0 \qquad \text{unit} = 1,\ldots,N_U;\ \text{chemical\_element} = 1,\ldots,N_E \tag{15}$$

Equations (12) are equality constraints for the solution. The solution method used is element-wise weighted total least squares. The weights are standard deviations of the solids flow rate measurements F^M, the analyses G^M and fraction m% M^M.

As before, the flow rates can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS). If there are no constraints, the minimal maximum norm solution can be calculated (LS, MinMax)[5].

2. Size fraction flow mass balance

Then the %m values of the sub-flows are solved. The equations are:

fraction m% measurements (16)

$$M_{flow,subflow} \approx M^{M}_{flow,subflow} \tag{16}$$

mass balance equations (17)

$$\sum_{flow=1}^{N_{F1}} e^{unit}_{flow} \cdot M^{M}_{flow,subflow} \cdot F_{flow,tot} = 0 \qquad \text{unit} = 1,\ldots,N_U \text{ unit not size reducing; subflow} = 1,\ldots,N_{SF} \tag{17}$$

sum fraction m% is a hundred (18)

$$\sum_{i=1}^{N_{SF}} M_{flow,subflow} = 100 \tag{18}$$

Equations (17) and (18) are equality constraints. The solution method used is element-wise total least squares and %m values can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS).

3. Bulk analyses

If the unsized bulk analyses are given, the balanced analyses are calculated as described above (see: unsized mass balance).

51.7.3. Sized by assay mass balance

Before a *sized by assay* mass balance solution, the *unsized* or *sized* mass balance must be solved first. The results $F_{flow,tot}$ of the *unsized* or *sized* solution are used in the *sized by assay* solution. In a *sized by assay* solution, the size fraction sub-flows are calculated first and after that the analyses are solved.

1. Sized by assay fraction sub-flows

The equations for size by assay sub-flow solutions are:

mass balance Equations (19) for each unit that is not size reducing

$$\sum_{flow=1}^{N_{F2}} e_{flow}^{unit} \cdot F_{flow,subflow} = 0 \quad \text{unit} = n_1,\ldots,n_{NU-1} \text{ unit not size reducing; subflow } = 1,\ldots,N_S \tag{19}$$

Above n_i is the index of the ith unit that is not size reducing. Units are indexed up to N_U - 1 to avoid linear dependency of equality constraints.

Sum of sub-flows is the total flow (20)

$$\sum_{subflow=1}^{NS} F_{flow,subflow} = F_{flow,tot} \quad \text{flow}=1,\ldots,N_{F2} \tag{20}$$

Analyses for each unit that are not size reducing (21)

$$\begin{aligned}&\sum_{flow=1}^{N_{F2}} e_{flow}^{unit} \cdot G_{flow,chemical_{element},subflow}^{M} \cdot F_{flow,subflow} \approx 0\\&\text{unit} = n_1,\ldots,n_{NU} \quad \text{unit not size reducing}\\&\text{chemical\_element} = 1,\ldots,N_E\\&\text{subflow} = 1,\ldots,N_S\end{aligned} \tag{21}$$

Fraction %m measurements (22)

$$\begin{aligned}&F_{flow,subflow} \approx M_{flow,subflow}^{M} * F_{flow,tot}/100\\&flow = 1,\ldots,N_{F2}\\&subflow = 1,\ldots,N_S\end{aligned} \tag{22}$$

or alternatively

flow measurements (23)

$$\begin{aligned}&F_{flow,subflow} \approx F_{flow,subflow}\\&flow = 1,\ldots,N_{F2}\\&subflow = 1,\ldots,N_S\end{aligned} \tag{23}$$

The solution method used is element-wise weighted least squares. The equations can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS).

2. Sized by assay fraction analyses

Let *G* and G^M be N_{F2*NS} *x* N_E matrices. The operator *vec* stacks the matrix columns into a vector.

The equations for the analyses solution are:

$$vec(G) \approx vec(G^M) \tag{24}$$

mass balance equations (25) for the analyses

$$vec(B * G) = 0 \tag{25}$$

where the $(N_S - 1)*(N_U - N_{SRU})x(N_{F2}*(N_S - 1))$ matrix B is defined

$$B = diag(B^{subflow}) \quad subflow = 1, \dots, N_S - 1 \tag{26}$$

Sub-flows are indexed up to N_S - 1 to avoid linear dependency of equality constraints. Where the operator *diag* adds matrices $B^{subflow}$ at the diagonal of the matrix B and

$$B^{subflow}_{unit,flow} = e^{unit}_{flow} * F_{flow,subflow} \tag{27}$$

the sum of sub-flows is the total flow (28)

$$\sum_{subflow=1}^{NS} F_{flow,subflow} \cdot G_{flow,chemical\_element,subflow} = F_{flow,tot} \cdot G_{flow,tot,chemical\_element} \tag{28}$$
$$flow = 1, \dots, N_{F2}$$
$$\text{chemical\_element} = 1,\dots,N_E$$

If the option minerals sum=100 is selected, the following Equations (29) are included:

$$\sum_{chemical\_element=1}^{N_E} G_{flow,chemical\_element,subflow} = 100 \qquad \text{flow} = 1,\dots,N_{F1} \quad subflow = 1, \dots, N_S \tag{29}$$

If Equations (29) are included, *B* is $(N_S - 1)*(N_U - N_{SRU} - 1)x(N_{F2}*(N_S - 1))$
to avoid linear dependencies.

The solution method used is weighted least squares. The equations can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS).

51.8. References

1. Golub, van Loan: Matrix Computations, Third edition 1996
2. Markovsky, Rastello, Premoli, Kuhush, van Huffel: The element-wise weighted total least squares problem, Computational Statistic & Data Analysis 50 (2006) pp.181-209
3. Lawson, Hanson: Solving Least Squares Problems, 1974
4. Haskell, Hanson: An Algorithm for Linear Least Squares Problems with Equality and Nonnegativity Constraints, Mathematical Programming 21 (1981), pp.98-118
5. Barrodale, Phillips: Algorithm 495, Solution of an Overdetermined System of Linear Equations in the Chebyshev Norm, ACM Transactions on Mathematical Software, Vol 1, No 3, September 1975, pp. 264-270.